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QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence.

DISCUSSIONS.

I. A REPLY TO PROFESSORS ZIWET AND JACKSON ON MECHANICS.[1]

BY EDWARD V. HUNTINGTON, Harvard University.

I cannot help being highly gratified at the favorable reception accorded my "Logical Skeleton of Elementary Dynamics"[2] by Professor ZIWET of Michigan and Professor JACKSON of Harvard, who say they regard it as "a really serviceable concrete scheme" by which "the whole approach to dynamical theory is extraordinarily simplified."

There are certain points, however, on which they confess they are still "unconvinced."

I. Professor JACKSON'S criticisms concern chiefly my treatment of "*mass*," which he regards, not as erroneous, but as inadequate. After some witty remarks about the proper draperies for skeletons like $F/F' = a/a'$ to wear, he contends that the "good old idea of mass" should be "searched for all there is in it." But, one asks at once, *which idea of mass does he mean?* For, as I have pointed out in my "Bibliographical Note on the Use of the Word Mass in Current Textbooks,"[3] there are at least four different senses in which the word mass is commonly used. Professor Jackson's preference, apparently, is to use the term in two of these senses at once, so that it shall convey simultaneously the idea of "inertia" (as measured by force over acceleration), and the idea of what he calls "gravitational power" (as measured by a beam balance). I do not believe that this double use of a hard-worked term like mass conduces to clearness; my reasons are set forth at length in the "Bibliographical Note" already cited (see especially Propositions F and G), and a reference to this note must here suffice.

II. Professor ZIWET'S criticisms cover a wider range.

For one thing, he points out an error on page 14, where I inadvertently stated a proposition about the resultant of a set of forces in three dimensions in a form applicable only to two dimensions; this I must acknowledge as an unfortunate blunder, which I beg the reader to correct.

Among Professor Ziwet's other criticisms, there are several which seem to indicate that certain passages in my "Skeleton" were not as clear as I had hoped to make them; and in a spirit of clarification rather than of controversy, I should like to set myself right in regard to some of these misunderstandings.

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[1] A. ZIWET, "Some Reflections on the Teaching of Mechanics, suggested by Professor E. V. Huntington's Article 'A Logical Skeleton of Elementary Dynamics'"; D. JACKSON, "Mass and Force in Elementary Dynamics," AMERICAN MATHEMATICAL MONTHLY, Vol. 24, pp. 296–297 and 298–300, June, 1917.

[2] AMERICAN MATHEMATICAL MONTHLY, Vol. 24, pp. 1–16, January, 1917.

[3] [This "Bibliographical Note" will be published in an early number of the MONTHLY. *Editors.*]

1. For example, referring to § 2 of Professor Ziwet's paper, I certainly did not mean to contend that "*force* as the active, aggressive principle" has any "logical priority" over "inert, passive *matter*."

In my opening section I meant, on the contrary, to set forth the fundamental concepts, "active forces" and "inert material bodies," as of absolutely coördinate importance, neither of them having any sort of priority over the other.

Professor Ziwet's misconception in regard to my intention is apparently due to a confusion on his part between the idea of "material body" on the one hand and the idea of the "mass of a body" on the other hand. Thus, in § 2, he identifies "mass" with "particle"; again, in § 5, he expressly uses "mass" as a synonym for "matter," "lump of matter," "body," etc. This particular confusion was one which I had especially hoped to guard against, by carefully avoiding, throughout the body of my paper, all use of the ambiguous word "mass"; the reappearance of the term "mass" in Professor Ziwet's discussion only goes to show how persistently intrusive a trouble-making little word may be.

It need hardly be said that a confusion of this sort absolutely vitiates all of Professor Ziwet's arguments in which the idea of the "mass of a body" occurs.

2. Again, judging from Professor Ziwet's remarks in § 4, I appear to have given him the impression that both of the "*lists of names of derived units*," tabulated on pp. 15–16, were regarded by me as of equal importance; whereas I had intended to make the contrast between these two tables as striking as possible.

Table II, which was placed in an appendix and carefully labelled "A useless complication still found in many text-books," contains names which are avowedly given as "horrible examples," like the *Lb.-ft*^{2} *per sec.*^{2} (!) as the unit of work, or the *Gm. per sec.*^{2}*-cm.* (!) as the unit of pressure.

Table I, on the other hand, was printed in the body of the text, and contains, as its heading indicates, *only names which are "actually used by engineers and pure scientists."* This Table I, which is so simple and obvious that there is nothing to "learn" about it, is the one that I advocate.

Professor Ziwet groups these two lists together quite indiscriminately,—sometimes condemning them equally as "a nuisance anyhow," which "the student should not be encouraged to consult," sometimes endorsing them equally, by saying that the student must be told that "both systems are in use and both are equally justifiable." Only as a sort of afterthought, at the end of his paper, does he really distinguish between them, and then he makes a statement which I confess I am at a loss to understand. He says, namely, that "the system of units here advocated is not used by a single writer on higher mechanics." This assertion seems to me to be quite unfounded, if "system of units" means the actual list of names of the units that I give in Table I; for all these names are the commonplace of all writers on the subject. For example, if you ask any physicist "What is an erg?" he will immediately reply: "an erg is a dyne-centimeter";

he would never think of saying: "an erg is a gram centimeter-square per second-square!"[1]

In my whole treatment of the matter of units, I had no thought of introducing anything new or strange; my aim was merely to record universally accepted terms in Table I, and to throw into the discard all superfluous complications (some of which Professor Ziwet apparently wishes to retain).

3. In this connection, I should like to say a word about Professor Ziwet's remark that "*it is a matter of personal opinion whether force or mass shall be regarded as the more fundamental; therefore why not introduce both these notions from the beginning, and use both in defining the derived units?*"

The possibility of carrying out this plan (which is often urged) *depends altogether on what one means by "mass."*[2] If (following the majority of writers) one means by "mass" the *inertia* of a body, as measured by force over acceleration, then it is not by any means a "matter of personal preference" whether force or mass shall be regarded as the more fundamental; obviously force *must* be put first, since inertia is defined in terms of it.[3] If, on the other hand, one means by mass something other than inertia, say the result of weighing on a beam balance, then indeed it may be possible to use both force and mass independently in defining the derived units—but only at the expense of frequent confusion between the units of inertia and the units of standard weight.

Everybody agrees, I believe, that the simple units in my Table I are adequate for purposes of measurement; why worry the student with more?

4. Returning to § 3 of Professor Ziwet's paper, in his discussion of my treatment of *acceleration* he has apparently overlooked the fact that my treatment of the rectilinear case was only introductory to a detailed treatment of the curvilinear case, with illustrative diagrams (page 7). My use of "fixed axes" in this connection, to which Professor Ziwet objects, appears to me both scientifically correct and pedagogically useful. It is surely of the utmost importance for the student to understand that the tangential and normal components of the acceleration of a particle are measured along axes which do *not* move with the particle.

5. Finally, the question of the early introduction of *vector notation*, and the question whether the study of dynamics should begin with *three dimensions or with two*, are important questions on which there may well be differences of opinion. I believe, however, that most teachers will agree with me that the historical order—proceeding from the simple to the more difficult—is the best.

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[1] Professor ZIWET says it seems "odd" to him to think (for instance) of the unit of kinetic energy in terms of force × distance; and yet this is precisely what is commonly done. For example, no less distinguished a writer than MAX PLANCK, in his special treatise on energy, expressly states that "energy is measured in units of work," that is, in units of force × distance.

[2] In Professor Ziwet's own treatise on Theoretical Mechanics (1906, p. 129), he explains the "mass of a body" merely as "an indestructible something underlying every physical body"; and although he soon speaks of multiplying mass by distance, velocity, etc., he nowhere explains how this "indestructible something" is to have a numerical measure assigned to it!

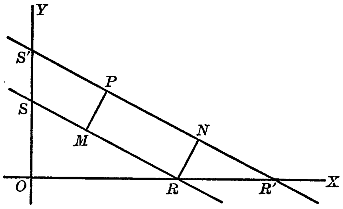
[3] This, by the way, is all that one means by "logical priority," which is apparently a new phrase to Professor Ziwet.

Other points in the papers of Professor Ziwet and Professor Jackson are met, I believe, by my discussion of "mass" in the "Bibliographical Note" already cited. I beg to repeat, however, that I do not put forward this discussion with any idea that it would be profitable for use in the class room. The chief end of a course in dynamics is not a definition of "mass," but the development of ability to master fundamental dynamical problems. *Any teacher who desires to avoid all these controversial matters, and proceed as rapidly as possible to the important principles,* has only to follow the program outlined in my "Logical Skeleton," from which all polemical matter has been intentionally excluded.[1]

II. RELATING TO THE DERIVATION OF A DISTANCE FORMULA.

BY R. M. MATHEWS, Junior College, Riverside, Calif.

In the usual course in analytic geometry the formula for the distance from a point to a line is derived from Hesse's "normal form" for the equation of a straight line. It is not necessary, however, to introduce the "normal form" merely for this purpose. The distance formula may easily be derived directly in the manner given below.



In the accompanying figure let (x_1, y_1) be the coördinates of the given point P and $ax + by + c = 0$ the equation of the given line SR. Also let $S'R'$ be a line through P parallel to SR and let RN be perpendicular to SR at R.

Then

$$d = PM = RN.$$

From similar triangles

$$\frac{RN}{RR'} = \frac{OS}{SR} \quad \text{and} \quad d = \frac{RR' \cdot OS}{SR}.$$

The equation of $S'R'$ is

$$ax + by + k = 0, \qquad \text{where} \qquad k = -ax_1 - by_1.$$

Also,

$$RR' = OR' - OR = -\frac{k}{a} + \frac{c}{a} = \frac{c-k}{a},$$

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[1] [Reprints of this "Skeleton," which a number of teachers have found useful in the class room, may be obtained from the Secretary, Professor W. D. Cairns, 27 King Street, Oberlin, O., at ten cents a copy. *Editors.*]